ELECTRICAL ENGINEERING

Communication Systems

Comprehensive Theory
with Solved Examples *and* Practice Questions

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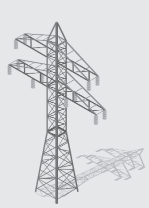
Communication Systems



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2

Amplitude Modulation

INTRODUCTION

In analog communication, message is analog and the carrier is sine wave, which is also analog in nature. The modulation techniques in analog communication can be classified into amplitude modulation (AM) and angle modulation techniques. The amplitude of the carrier signal is varied in accordance with the message to obtain modulated signal in case of amplitude modulation.

After studying the theory of amplitude modulation techniques, one will be able to know that an AM wave is made of a number of frequency components having a specific relation to one another. Based on this observation, AM can be further classified as double sideband full carrier (DSBFC), double sideband suppressed carrier (DSBSC), single sideband (SSB) and vestigial sideband (VSB) modulation techniques. This is based on how many components of the basic amplitude modulated signal are chosen for transmission. This is followed by a description of different methods for the generation of AM, DSBSC, SSB and VSB signals. To summarize, this chapter describes the basic essence of all the amplitude modulation techniques. In this chapter AM and its variants, their differences, merits and demerits are discussed. The students will also be able to calculate the frequencies present, plot the spectrum, the power or current associated with different frequency components and finally bandwidth requirements.

2.1 AMPLITUDE MODULATION

Amplitude modulation is the process of changing the amplitude of a relatively high frequency carrier signal in proportion with the instantaneous value of the modulating signal (information).

Amplitude modulation is a relatively inexpensive, low quality form of modulation that is used for commercial broadcasting of both audio and video signals.

Consider a sinusoidal carrier wave $c(t)$ defined by

$$c(t) = A_c \cos(2\pi f_c t)$$

where the peak value A_c, is called the *carrier amplitude* and f_c is called the *carrier frequency*. For convenience, we have assumed that the phase of the carrier wave is zero. It is justified in making this assumption since the carrier source is always independent of the message source. We refer to $m(t)$ as the message signal which is baseband in nature. **Amplitude modulation is defined as a process in which the amplitude of the carrier wave $c(t)$ is varied linearly with the message signal $m(t)$ keeping other parameters constant.**

Amplitude modulation is linear process but AM modulators are non-linear devices.

2.1.1 Time-Domain Description

The standard form of an amplitude-modulated (AM) wave is defined by

$$s(t) = A_c[1 + k_a m(t)] \cos(2\pi f_c t)$$

where k_a is a constant called the *amplitude sensitivity* of the modulator. The modulated wave so defined is said to be a "standard" AM wave, because its frequency content is *fully* representative of amplitude modulation.

- The amplitude of the time function multiplying $\cos(2\pi f_c t)$ is called the *envelope* of the AM wave $s(t)$. Using $a(t)$ to denote this envelope, we may thus write

$$a(t) = A_c\left|1 + k_a m(t)\right| \qquad \text{...(i)}$$

- Two cases of particular interest arise, depending on the magnitude of $k_a m(t)$, compared to unity.
- For case 1, we have

$$\left|k_a m(t)\right| \le 1, \text{ for all } t$$

Under this condition, the term $1 + k_a m(t)$ is always nonnegative. We may therefore simplify the expression for the envelope of the AM wave by writing

$$a(t) = A_c[1 + k_a m(t)], \quad \text{for all } t$$

- For case 2, on the other hand, we have

$$\left|k_a m(t)\right| > 1, \text{ for all } t$$

Under this condition, we must use equation (i) for evaluating the envelope of AM wave.

The maximum absolute value of $k_a m(t)$ multiplied by 100 is referred to as the percentage modulation.

> **NOTE**
> The envelope of the AM wave has a waveform that bears a *one-to-one correspondence* with that of the message signal if and only if the percentage modulation is less than or equal to 100%. This correspondence is destroyed if the percentage modulation exceeds 100%. In the later case, the modulated wave is said to suffer from **envelope distortion**, and the wave is said to be **over modulated**.

The complexity of the detector (i.e., the demodulation circuit used to recover the message signal from the incoming AM wave at the receiver) is greatly simplified if the transmitter is designed to produce an envelope $a(t)$ that has the same shape as the message signal $m(t)$.For this, two conditions are need to be satisfied.

1. The percentage modulation should be less than 100%, so as to avoid envelope distortion.
2. The message bandwidth, f_m, should be small as compared to the carrier frequency f_c, so that the envelope $a(t)$ may be visualized satisfactorily. Here, it is assumed that the spectral content of the message signal is negligible for frequencies outside the interval $-f_m \le f \le f_m$, i.e., message signal is basedband in nature.

2.1.2 Observations

1. The frequency of the sinusoidal carrier is much higher than that of the modulating signal.
2. In AM, the instantaneous amplitude of the sinusoidal high frequency carrier is changed in proportion to the instantaneous amplitude of the modulating signal. This is the principle of AM.
3. The time domain display of AM signal is as shown in figure. This AM signal is transmitted by a transmitter. The information in the AM signal is contained in the amplitude variations of the carrier of the envelope shown by dotted lines .
4. Note that the frequency and phase of the carrier remain constant.
5. AM is used in the applications such as radio transmission, TV transmission.

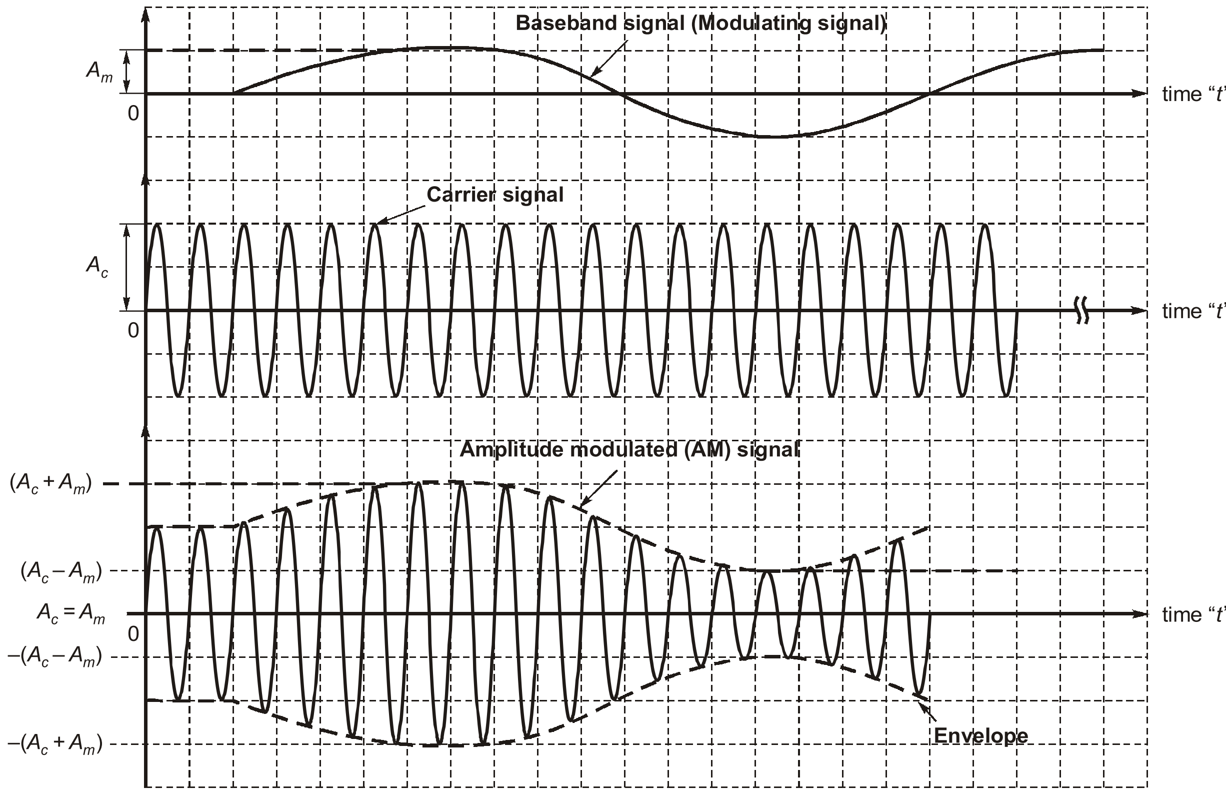


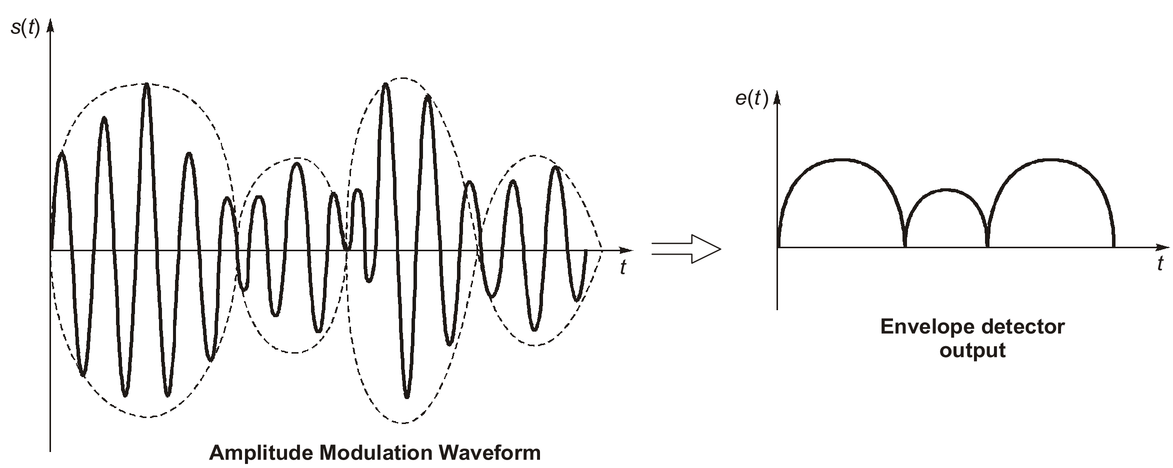
Figure : *AM waveform for sinusoidal modulating signal*

EXAMPLE : 2.1

The amplitude modulated waveform $s(t) = A_c[1 + K_a m(t)] \cos \omega_c t]$ is fed to an ideal envelope detector. The maximum magnitude of $K_a m(t)$ is greater than 1. Which of the following could be the detector output?

(a) $A_c m(t)$ (b) $A_c^2[1 + K_a m(t)]^2$ (c) $\left[A_c \left|1 + K_a m(t)\right|\right]$ (d) $A_c \left|1 + K_a m(t)\right|^2$

Solution : (c)



$$s(t) = A_c[1 + K_a m(t)] \cos \omega_c t$$

Since, $K_a m(t) > 1$, hence the modulation index of the AM wave is greater than unity. Hence, the AM waveform is overmodulated. Consider $m(t) = A_m \sin \omega_m t$, the AM wave and the envelope detector output can be drawn as shown above:

The envelope detector output for the given overmodulated AM wave can be written mathematically as

$$e(t) = |A_c(1+K_a m(t))| = A_c|1+K_a m(t)|$$

2.1.3 Frequency Domain Description

To develop the frequency description of the AM wave, we take the Fourier transform of both sides. Let $S(f)$ denote the Fourier transform of $s(t)$, and $M(f)$ denote the Fourier transform of the message signal $m(t)$; we refer to $M(f)$ as the message spectrum. Accordingly, using the Fourier transform of the cosine function $A_c \cos(2\pi f_c t)$ and the frequency-shifting property of the Fourier transform, we may write

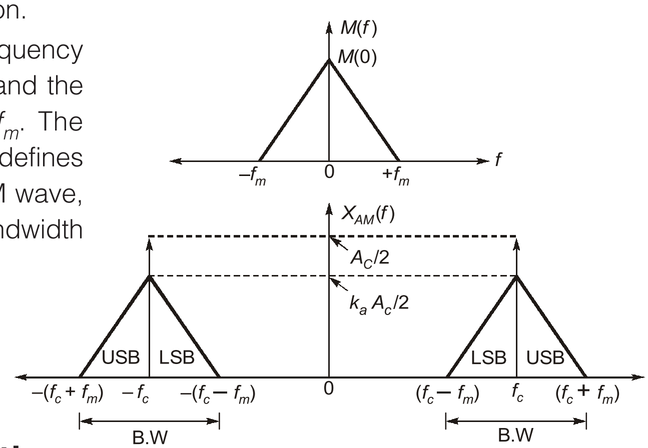
$$S(f) = \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)] + \frac{k_a A_c}{2}[M(f - f_c) + M(f + f_c)]$$

Let the message signal $m(t)$ be band-limited to the interval $-f_m \leq f \leq f_m$, the shape of the spectrum shown in this figure is intended for the purpose of illustration only.

The spectrum consists of two delta functions weighted by the factor $A_c/2$ and occurring at $\pm f_c$, and two versions of the message spectrum translated in frequency by $\pm f_c$ and scaled in amplitude by $kaA_c/2$.

1. For positive frequencies, the portion of the spectrum of the modulated wave lying above the carrier frequency f_c is called the **upper sideband**, whereas the symmetric portion below f_c is called the **lower sideband**. For negative frequencies, the image of the upper sideband is represented by the portion of the spectrum below $-f_c$ and the image of the lower sideband by the portion above $-f_c$. The condition $f_c > W$ ensures that the sidebands do not overlap. Otherwise, the modulated wave exhibits spectral overlap and therefore frequency distortion.
2. For positive frequencies, the highest frequency component of the AM wave is $f_c + f_m$, and the lowest frequency component is $f_c - f_m$. The difference between these two frequency defines the transmission bandwidth B for an AM wave, which is exactly twice the message bandwidth f_m; that is

   $$\text{B.W} = (f_c + f_m) - (f_c - f_m)$$
   $$\text{B.W} \approx 2 f_m \text{ Hz or kHz}$$
   $$\text{B.W} = 2\omega_m \text{ rad/sec}$$



2.1.4 Single-Tone and Multi-Tone Modulation

1. **Single-Tone modulation:** If the message signal have single frequency, then the corresponding modulation is called as single-tone modulation. Eg:- $m(t) = A_m \cos 2\pi f_m t$.

   $$m(t) = A_m \cos 2\pi f_m t; \qquad c(t) = A_c \cos 2\pi f_c t$$

   Single tone AM modulated signal, $X_{AM}(t) = A_c(1 + K_a A_m \cos 2\pi f_m t)\cos 2\pi f_c t$

   Here $\mu = m_a = K_a A_m \rightarrow$ Modulation index, $X_{AM}(t) = A_c(1 + m_a \cos 2\pi f_m t)\cos 2\pi f_c t$
2. **Multi-Tone modulation:** If the message signal contains multiple frequencies, then the corresponding modulation is called as multi-tone modulation. Eg:- $A_{m1}\cos 2\pi f_{m1} t + A_{m2}\cos 2\pi f_{m2} t$.

2.2 SINGLE TONE AMPLITUDE MODULATION

$$c(t) = A_C \cos \omega_c t \text{ ....carrier signal}$$
$$m(t) = A_m \cos \omega_m t \text{ .... modulating signal}$$

then after modulation, we get

$$\therefore \quad X_{AM}(t) = [A_c + A_m \cos \omega_m t] \cos \omega_c t$$

$$X_{AM}(t) = A_c \left[1 + \frac{A_m}{A_c} \cos \omega_m t\right] \cos \omega_c t$$

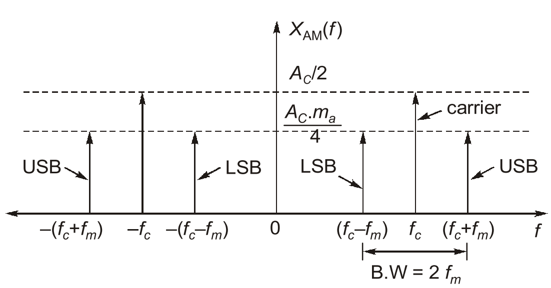
$$X_{AM}(t) = A_c [1 + m_a \cos \omega_m t] \cos \omega_c t$$

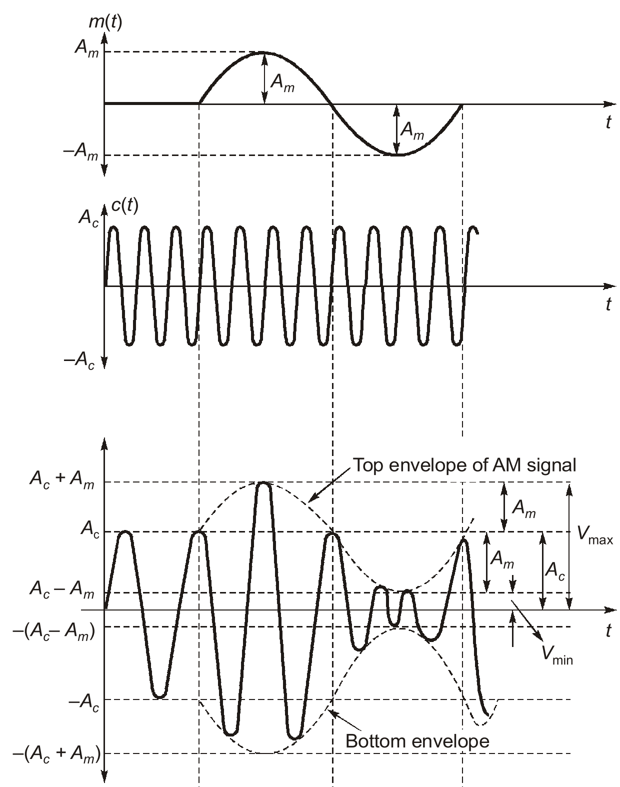
where, $m_a = \dfrac{A_m}{A_c}$ = Modulation Index or Depth of modulation.

The above equation can also be written as

$$X_{AM}(t) = \underbrace{A_c \cos \omega_c t}_{\text{Full carrier}} + \frac{1}{2} m_a A_c \underbrace{\cos(\omega_c + \omega_m)t}_{\text{USB}} + \frac{1}{2} m_a A_c \underbrace{\cos(\omega_c - \omega_m)t}_{\text{LSB}}$$

2.2.1 Spectrum of Sinusoidal AM signal





$$2 A_m = V_{max} - V_{min}$$

$$\Rightarrow \quad A_m = \frac{V_{max} - V_{min}}{2}$$

$$A_c = V_{max} - A_m = V_{max} - \frac{V_{max} - V_{min}}{2} = \frac{V_{max} + V_{min}}{2}$$

Finally we get, $$\mu = m = \frac{A_m}{A_C} = \frac{V_{max} - V_{min}}{V_{max} + V_{min}} \rightarrow \text{modulation index}$$

$$V_{max} = A_c + A_m = A_c[1 + m_a] \rightarrow \text{Peak value of envelope}$$

$$V_{min} = V_{max} - 2A_m = A_c + A_m - 2A_m = A_c - A_M$$

$$V_{min} = A_c[1 - m_a] \rightarrow \text{Minimum value of envelope}$$

- % modulation = $m_a \times 100$
- Modulation index gives the depth to which the carrier signal is modulated.
- For $m(t)$ to be preserved in the envelope of AM signal, $m_a \leq 1$
  i.e., $A_m \leq A_c$
  so, range of m_a is, $0 \leq m_a \leq 1$

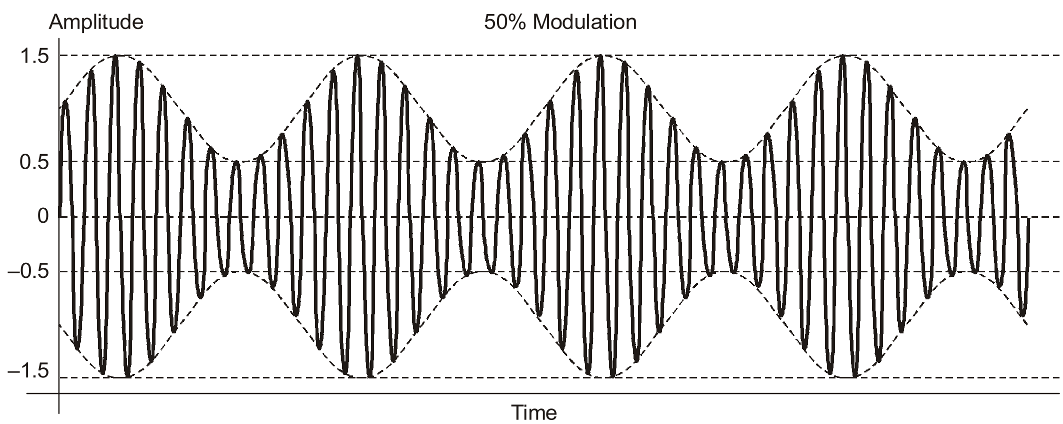
2.2.2 Types of AM Based on the Value of m_a (Modulation Index)

Based on the value of m_a, AM have three types :

(1) Under modulation ($m_a < 1$) (2) Critical modulation ($m_a = 1$) (3) Over modulation ($m_a > 1$)

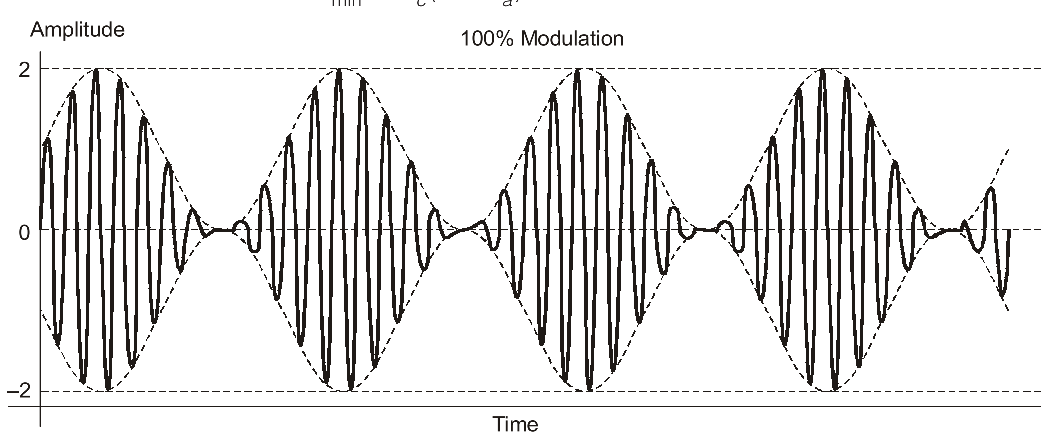
1. **Under Modulation ($m_a < 1$):** When $m_a < 1$, i.e., $A_M < A_c$ or $A_m < 1$, under modulation takes place.

$$V_{min} = A_c(1 - m_a) \Rightarrow +ve$$



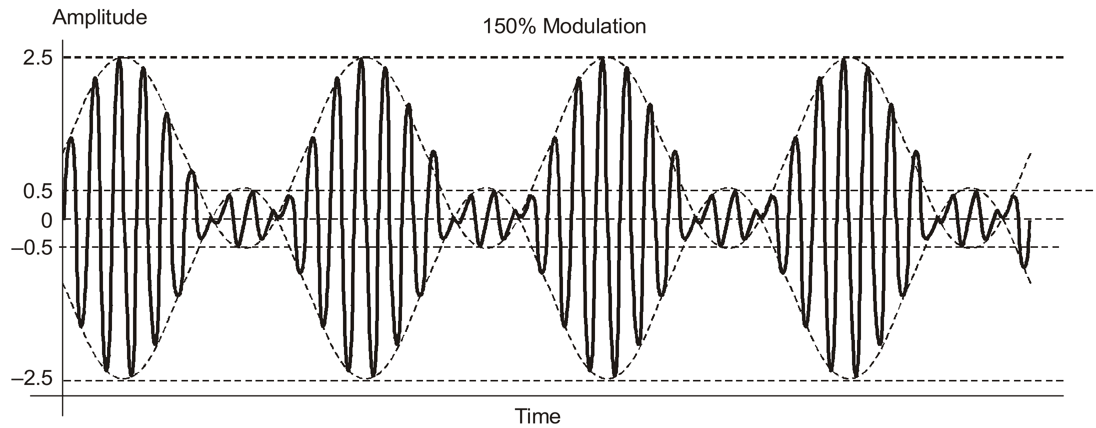
2. **Critical Modulation ($m_a = 1$):** When $m_a = 1$, i.e., $A_m = A_c$ critical modulation takes place

$$V_{min} = A_c(1 - m_a) = 0$$



3. **Over Modulation ($m_a > 1$):** When $m_a > 1$, i.e., $A_m > A_c$ or $A_m > 1$, over modulation takes places and the signal gets distorted. Because, the negative part of waveform gets cut from the waveform leaving behind a "square wave type" of signal, which generates infinite harmonics as "non-linear distortion" or "envelope distortion".

$$V_{min} = A_c(1 - \mu) \Rightarrow -ve$$



2.3 POWER RELATIONS IN AM

- In practice, the AM wave is a voltage or current wave.
- An AM wave consists of carrier and two sidebands. Hence the AM wave will contain more power than the power-contained by an unmodulated carrier.
- The amplitudes of the two sidebands are dependent on the modulation index "m_a". Hence the power contained in the sidebands depends on the value of m_a. Hence the total power in an AM wave is a function of the value of modulation index m_a.

2.3.1 The Total Power in AM

The total power in an AM wave is given by,

$$P_t = [\text{Carrier Power}] + [\text{Power in USB}] + [\text{Power in LSB}]$$

$$\therefore \quad P_t = \frac{E^2}{R} + \frac{E_{USB}^2}{R} + \frac{E_{LSB}^2}{R}$$

Where E, E_{USB} and E_{LSB} are the RMS values of the carrier and sideband amplitudes and R is the characteristic resistance of antenna in which the total power is dissipated.

2.3.2 Carrier Power (P_c)

The carrier power is given by, $P_c = \dfrac{E^2}{R} = \dfrac{\left[A_c/\sqrt{2}\right]^2}{R} = \dfrac{A_c^2}{2R}$

2.3.3 Power in the Sidebands

- The power in the two sidebands is given as $P_{USB} = P_{LSB} = \dfrac{E_{SB}^2}{R}$
- As we know the peak amplitude of each sideband is $\dfrac{m_a A_c}{2}$

$$P_{USB} = P_{LSB} = \frac{\left[m_a A_c / 2\sqrt{2}\right]^2}{R} = \frac{m_a^2 A_c^2}{8R}$$

∴ $$P_{USB} = P_{LSB} = \frac{m_a^2}{4} \times \frac{A_c^2}{2R}$$

Hence $$P_{USB} = P_{LSB} = \frac{m_a^2}{4} P_c$$

2.3.4 Total Power

Therefore the total power is given by

$$P_t = P_c + P_{USB} + P_{LSB} = P_c + \frac{m_a^2}{4} P_c + \frac{m_a^2}{4} P_c$$

∴ $$\boxed{P_t = \left[1 + \frac{m_a^2}{2}\right] P_c} \; W$$

For general message signal

Total power is given by $P_t = P_c + P_{SB}$; $P_t = P_c + \frac{1}{2} P_m$

where $P_m \rightarrow$ Power of message signal.

In case of sinusoidal message signal $P_m = \frac{A_m^2}{2}$

$$P_t = \frac{A_c^2}{2} + \frac{1}{2}\left(\frac{A_m^2}{2}\right) = \frac{A_c^2}{2}\left[1 + \left(\frac{A_m}{A_c}\right)^2\right] = P_c\left[1 + \frac{(m_a)^2}{2}\right]$$

For sinusoidal message signal.

2.3.5 Peak Envelope Power

Peak envelope value $V_{max} = A_c(1 + m_a)$

Peak envelope power $= A_c^2(1 + m_a)^2$

2.3.6 Transmission Efficiency

- Since the information about the message signal is only contained in sidebands, transmission efficiency of an AM wave is the ratio of the transmitted power which contains the information (i.e. the total sideband power) to the total transmitted power.

∴ $$\eta = \frac{P_{LSB} + P_{USB}}{P_t} = \frac{\left[\frac{m_a^2}{4} P_c + \frac{m_a^2}{4} P_c\right]}{\left[1 + \frac{m_a^2}{2}\right] P_c} = \frac{m_a^2 / 2}{1 + \frac{m_a^2}{2}} = \frac{m_a^2}{2 + m_a^2}$$

- The percentage transmission efficiency is given by

$$\eta\% = \frac{m_a^2}{2 + m_a^2} \times 100\%$$

- A lot of power is wasted in the transmission of carrier signal, which doesn't contain any information about the message signal.

2.3.7 AM Power in Terms of Current

- The total power P_t of an AM wave and the carrier power P_c can be expressed in terms of currents.
- Assume I_c to be the RMS current corresponding to the unmodulated carrier and I_t to be the RMS current for AM wave.

$$P_c = I_c^2 R \quad \text{and} \quad P_t = I_t^2 R$$

$$\therefore \quad \frac{P_t}{P_c} = \frac{I_t^2}{I_c^2} \times \frac{R}{R} = \left[\frac{I_t}{I_c}\right]^2$$

But
$$\frac{P_t}{P_c} = \left[1+\frac{m_a^2}{2}\right] \Rightarrow \left[\frac{I_t}{I_c}\right]^2 = 1+\frac{m_a^2}{2} \Rightarrow I_t = I_c\left[1+\frac{m_a^2}{2}\right]^{1/2}$$

The similar equations are valid in terms of voltage also, i.e., $V_t = V_c\left[1+\frac{m_a^2}{2}\right]^{1/2}$.

EXAMPLE : 2.2

An AM signal with a carrier of 1 kW has 200 Watts in each sideband. What is the percentage of modulation?

Solution:

$$P_c = 1000 \text{ W},$$
$$P_{\text{USB}} = P_{\text{LSB}} = 200 \text{ W}$$
$$\therefore \quad \text{Total power } P_t = 1000 + 200 + 200 = 1400 \text{ W}$$
$$P_t = P_c\left[1+\frac{m^2}{2}\right]$$
$$\therefore \quad 1400 = 1000\left[1+\frac{m^2}{2}\right]$$
$$\therefore \quad \text{Percentage modulation, } m = 0.8944 \text{ or } 89.44\%$$

2.4 MODULATION BY A MULTIPLE SINGLE TONE SIGNALS (MULTI-TONE MODULATION)

- Until now we have assumed that only one modulating signal is present. But in practice more than one modulating signals will be present. Let us see first how to express the AM wave when more than one modulating signals are simultaneously used.
- Let us assume that there are two modulating signals.

$$m_1(t) = A_{m1}\cos\omega_{m1}t$$
$$m_2(t) = A_{m2}\cos\omega_{m2}t$$

$\therefore$ The instantaneous value of the envelope of AM wave is

$$a(t) = A_c + m_1(t) + m_2(t)$$

Substituting the value of $a(t)$ in the standard AM equation, we get,

$$S_{\text{AM}}(t) = A_c\left[1+\frac{A_{m1}}{A_c}\cos\omega_{m1}t + \frac{A_{m2}}{A_c}\cos\omega_{m2}t\right]\cos\omega_c t$$

$$= \left[A_c^2 + \left(\frac{1}{1+t^2}\right)^2 + \frac{2A_c}{1+t^2} + \frac{t^2}{(1+t^2)^2}\right]^{1/2}$$

$$= \left[A_c^2 + \frac{1}{1+t^2} + \frac{2A_c}{1+t^2}\right]^{1/2}$$

$$= A_c\left[1 + \frac{2}{A_c(1+t^2)} + \frac{1}{A_c^2(1+t^2)}\right]^{1/2}$$

↓ neglect

$$[x(t)]_{env} = A_c\left[1 + \frac{2}{A_c(1+t^2)}\right]^{1/2}$$

Using exp.

$$[x(t)]_{env} = A_c\left[1 + \frac{1}{2}\frac{2}{A_c(1+t^2)}\right] = A_c + \frac{1}{1+t^2}$$

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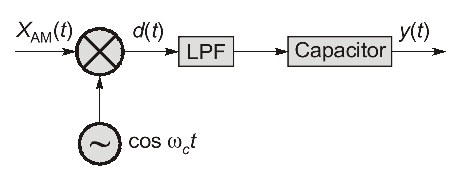
OBJECTIVE BRAIN TEASERS

Q.1 Total power saving when carrier and one of the sidebands are suppressed in an AM wave modulated to a depth of 50% is
(a) 66.67% (b) 83.33%
(c) 94.44% (d) 100%

Q.2 For a message signal $m(t)$ = 10 cos 100 t with 60% modulation, the maximum envelope time will be
(a) 10.3 ms (b) 13.3 ms
(c) 33.3 ms (d) 10 ms

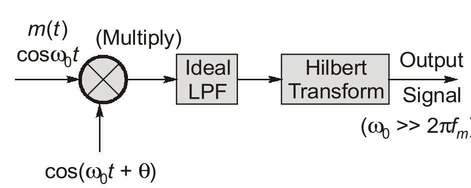
Q.3 The modulation index of an AM wave is changed from 0 to 1. The transmitted power is
(a) unchanged
(b) halved
(c) doubled
(d) increased by 50 percent

Q.4 For given synchronous demodulator can demodulate AM signal $X_{AM}(t) = [A + m(t)]\cos\omega_c t$. The value of $y(t)$ is



(a) $m(t)$ (b) $\frac{m(t)}{2}$
(c) $\frac{m(t)}{4}$ (d) zero

Q.5 A message $m(t)$ band limited to the frequency f_m has a power of P_m. The power of output signal is



(a) $\frac{P_m\cos\theta}{2}$ (b) $\frac{P_m}{4}$
(c) $\frac{P_m\sin^2\theta}{4}$ (d) $\frac{P_m\cos^2\theta}{4}$

ANSWERS KEY

1. (c) 2. (b) 3. (d) 4. (b) 5. (c)

HINTS & EXPLANATIONS

1. (c)

$$\% \text{ Power saving} = \frac{P_C + P_{USB}}{P_t} \times 100\%$$

$$= \frac{P_C + \frac{P_C \mu^2}{4}}{P_C\left(1+\frac{\mu^2}{2}\right)} \times 100\% = \frac{\mu^2+4}{2(2+\mu^2)} \times 100\%$$

$$= \frac{4+0.25}{2(2+0.25)} \times 100\% = 94.44\%$$

2. (b)

For the envelope detector output to follow the envelope of AM signals,

$$RC \le \frac{1}{\omega_m} \cdot \frac{\sqrt{1-\mu^2}}{\mu}$$

For $m(t) = 10\cos 100t$, $\omega_m = 100$ rad/sec

$$\therefore \quad RC \le \frac{1}{100} \times \frac{\sqrt{1-(0.6)^2}}{0.6}$$

$$RC \le \frac{1}{100} \times \frac{0.8}{0.6}$$

$$RC \le 13.33 \text{ ms}$$

3. (d)

For AM wave, transmitted power is

$$P_t = P_C\left(1+\frac{\mu^2}{2}\right)$$

For $\mu = 0$, $P_{t_1} = P_C$

For $\mu = 1$, $P_{t_2} = \frac{3P_C}{2}$

The percentage change in transmitted power,

$$= \frac{P_{t_2} - P_{t_1}}{P_{t_1}} \times 100\% = \frac{\frac{3}{2}P_C - P_C}{P_C} \times 100\% = 50\%$$

Hence, the transmitted power is increased by 50 percent when μ is changed from 0 t to 1.

4. (b)

We have,

$$d(t) = [A + m(t)]\cos\omega_c t \cdot \cos\omega_c t$$

$$d(t) = [A + m(t)]\cos^2 \omega_c t$$

$$d(t) = [A + m(t)]\frac{(1+\cos 2\omega_c t)}{2}$$

$$d(t) = \frac{A}{2} + \frac{A}{2}\cos 2\omega_c t + \frac{m(t)}{2} + \frac{m(t)\cdot\cos 2\omega_c t}{2}$$

On passing through low pass filter,

$$y'(t) = \frac{A}{2} + \frac{m(t)}{2}$$

The capacitor blocks the DC component

Hence, $y(t) = \frac{m(t)}{2}$

5. (c)

The input to the low pass filter,

$$s(t) = m(t)\cos\omega_o t \cdot \cos(\omega_o t + \theta)$$

$$s(t) = \frac{m(t)}{2}[\cos(2\omega_o t + \theta)\cos\theta]$$

$$s(t) = \underbrace{\frac{m(t)\cdot\cos(2\omega_o t + \theta)}{2}}_{\substack{\text{Frequency range from} \\ 2f_o - f_m \text{ to } 2f_o + f_m \\ \text{where } f_o = \omega_o/2\pi}} + \underbrace{\frac{m(t)\cos\theta}{2}}_{\substack{\text{Frequency} \\ \text{range from} \\ 0 \text{ to } f_m}}$$

Hence, the output of LPF,

$$y'(t) = \frac{m(t)\cdot\cos\theta}{2}$$

The Hilbert transform of $y'(t)$ gives the output signal as

$$y(t) = \frac{m(t)\cdot\sin\theta}{2}$$

(Hilbert transform is a 90° phase shifter).

Hence, the power of output signal is

$$P_o = \frac{P_m \sin^2\theta}{4}$$

■■■■

CONVENTIONAL BRAIN TEASERS

Q.1 The input to an envelope detector is a single-tone AM signal

$$x_{AM}(t) = A[1 + m_a \cos(\omega_m t)]\cos(\omega_c t)$$

where m_a is constant, $0 < m_a < 1$, and $\omega_c >> \omega_m$.

(i) Show that if the detector output is to follow the envelope of $x_{AM}(t)$, it requires that at any time t_o

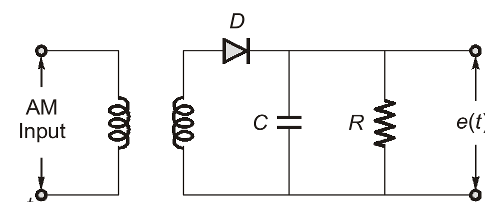
$$\frac{1}{RC} \geq \omega_m \left(\frac{m_a \sin\omega_m t_o}{1 + m_o \cos\omega_m t_o} \right)$$

(ii) Also prove if the detector output is to follow the envelope at all times, it is required that

$$RC \leq \frac{1}{\omega_m} \frac{\sqrt{1 - m_a^2}}{m_a}$$

1. (Sol.)

(i) The input to an envelope detector is given as

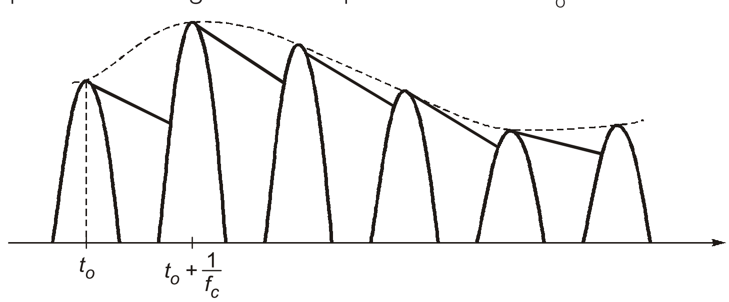


$$x_{AM}(t) = A[1 + m_a \cos(\omega_m t)] \cdot \cos(\omega_c t)$$

The maximum amplitude of the signal, at $t = t_o$

$$V_o = A + m_a A \cos(\omega_m t_o)$$

Assume the capacitor discharges from the peak value at $t = t_o$.



The voltage $v_c(t)$ across the capacitor,

$$V_c(t) = V_o e^{-(t - t_o)/Rc}$$

Rate of discharge, $$\frac{-dV_c(t)}{dt} = \frac{V_o}{RC} e^{-(t - t_o)/RC}$$

Rate of change at $t = t_o$,

$$\left(-\frac{dV_c(t)}{dt} \right)_{t = t_o} = \frac{V_o}{RC} = \frac{A}{RC}(1 + m_a \cos\omega_m t_o) \qquad \text{...(i)}$$

Now, the rate of change of envelope of AM signal,

$$-\frac{de(t)}{dt} = -\frac{d}{dt}[A(1 + m_a \cos\omega_m t)]$$

$$-\frac{de(t)}{dt} = \omega_m m_a A \sin\omega_m t$$

At $t = t_o$, $$\left(-\frac{de(t)}{dt} \right)_{t = t_o} = \omega_m \cdot m_a \cdot A \sin\omega_m t_o \qquad \text{...(ii)}$$